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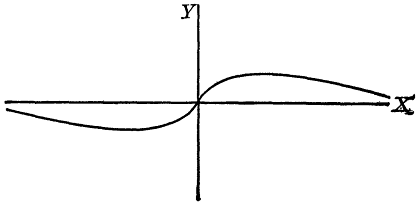
that is, the revolution of an infinite area gives a finite volume. Again, the ordinate of the center of gravity of this area is

$$\bar{y} = \frac{\int_0^\infty dx \int_0^{a^2x/(x^2+a^2)} y\,dy}{\int_0^\infty dx \int_0^{a^2x/(x^2+a^2)} dy} = \frac{\frac{\pi a^3}{8}}{\infty} = 0;$$

that is, although the area lies entirely above the x-axis and is indefinitely large,



its center of gravity is indefinitely close to that axis.

By the theorem of Pappus we have

$$2\pi\bar{y}A = V,$$

whence we have an indeterminate form evaluated

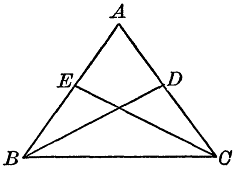
$$2\pi(0)\infty = \frac{\pi^2a^3}{4}.$$

II. Relating to a Demonstration of a Geometrical Theorem.

By William E. Heal, Washington, D. C.

If the bisectors of two angles of a triangle are equal, the triangle is isosceles.

This is a very celebrated theorem and has been demonstrated in many ways. It was proposed in the Monthly as problem 42 and several demonstrations were published in Vol. II, pp. 157 and 189–191. Of these proofs all but one were by indirect methods; that is, by use of the *reductio ad absurdum.* The following direct method of proof was communicated to the writer some months ago by Dr. Artemas Martin of the U. S. Coast Survey and seems to be eminently worthy of preservation.



If $BD = CE$ we are to prove that $AB = AC$.

We have by a well-known theorem

$$AB \times BC = (AE + BE)BC = BD^2 + AD \times DC, \tag{1}$$

$$AC \times BC = (AD + DC)BC = CE^2 + AE \times EB. \tag{2}$$

If $BD = CE$ these become

$$(AE + BE)BC = BD^2 + AD \times DC, \tag{3}$$

$$(AD + DC)BC = BD^2 + AE \times EB. \tag{4}$$

Subtracting (3) from (4) and transposing,

$$AE \times BC + BE \times BC + AE \times BE = AD \times DC + AD \times BC + DC \times BC. \quad (5)$$

We also have

$$AB : BC = AD : DC,$$

$$AC : BC = AE : EB,$$

$$\therefore \quad BC \times AD = AB \times DC = (AE + BE)DC, \quad (6)$$

$$BC \times AE = AC \times EB = (AD + DC)EB. \quad (7)$$

Substituting these values of $BC \times AD$ and $BC \times AE$ in (5) and dropping $BE \times CD$ from both sides we have

$$AD \times EB + BC \times EB + AE \times EB = AD \times DC + AE \times DC + BC \times DC.$$

Or

$$(AD + BC + AE)EB = (AD + AE + BC)DC.$$

$$\therefore \quad EB = DC.$$

Substituting $EB = DC$ in (5) we get, after dropping $EB \times BC$ from both sides,

$$AE \times BC + AE \times BE = AD \times EB + AD \times BC$$

or

$$(BC + BE)AE = (BC + BE)AD.$$

Hence

$$AE = AD.$$

Therefore

$$AB = AC.$$

III. Concerning Huntington's *Continuum and Other Types of Serial Order*.[1]

By Lester S. Hill, Princeton University.

If A_1 is any subclass of an ordered class A, then all elements a of A which are ordinally less than every element of A_1 ($a < A_1$) constitute a lower segment of A; and all elements $a > A_1$ constitute an end segment. If A_1 and A_2 are two subclasses with $A_1 < A_2$ then all elements a for which $A_1 < a < A_2$ constitute a mid segment. Of course A_1 or A_2, or each of them, may reduce to a single element of A.

The order type of A is continuous if

(D) A is dense: between every two elements of A lie other elements;

(G) A is gap-free: If A' is a lower segment, A'' an end segment, and $A = A' + A''$, then A' has a last element or else A'' has a first. ($A = A' + A''$ implies $A' < A''$.)

The ordered class A of real numbers satisfies (D), (G) and also

(F_0) A is denumerably framed: A contains a *denumerable* subclass B which is dense in A; between every two elements of A lie elements of B.

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[1] See Book Review on page 325 of this issue.